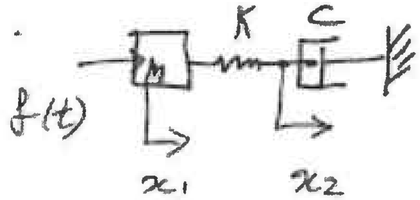
Q1.



(a)

$f(t) \rightarrow \boxed{m} \leftarrow K(x_1 - x_2)$

$K(x_2 - x_1)$ $\leftarrow$ $\dot{x}_2$ $\leftarrow$ $C(\dot{x}_2 - 0)$

$\Sigma F = m\ddot{x}_1$ $\qquad\qquad$ $\Sigma F = 0$

$-K(x_1 - x_2) + f(t) = m\ddot{x}_1$ $\quad$ Eq(1) $\quad$ 2

$-K(x_2 - x_1) - C\dot{x}_2 = 0$ $\quad$ Eq(2) $\quad$ 2

$m\ddot{x}_1 + K(x_1 - x_2) = f(t)$ $\quad$ ① $\quad$ 2

$K(x_2 - x_1) + C\dot{x}_2 = 0$ $\quad$ ② $\quad$ 2 $\quad$ [8]

(b) Take Laplace Transform

$m s^2 X_1(s) + K\left(X_1(s) - X_2(s)\right) = F(s)$ $\quad$ 1 $\quad$ 1

$K\left(X_2(s) - X_1(s)\right) + Cs X_2(s) = 0$ $\quad$ 1 $\quad$ 2

$$\begin{bmatrix} ms^2+k & -k \\ -k & k+cs \end{bmatrix} \begin{bmatrix} X_1(s) \\ X_2(s) \end{bmatrix} = \begin{bmatrix} F(s) \\ 0 \end{bmatrix} \qquad 1$$

$$\begin{bmatrix} X_1(s) \\ X_2(s) \end{bmatrix} = \begin{bmatrix} ms^2+k & -k \\ -k & k+cs \end{bmatrix}^{-1} \begin{bmatrix} F(s) \\ 0 \end{bmatrix} \qquad 1$$

$$\begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \frac{\begin{bmatrix} k+cs & k \\ k & ms^2+k \end{bmatrix} \begin{bmatrix} F \\ 0 \end{bmatrix}}{(ms^2+k)(k+cs)-k^2} \qquad 1$$

$$X_2(s) = \frac{k\,F(s)}{mks^2 + mcs^3 + \cancel{k^2} + kcs - \cancel{k^2}} \qquad 1$$

$$\frac{X_2}{F} = \frac{k}{s(mcs^2 + mks + kc)} \qquad 2$$

[8]

$$\frac{X_2}{F} = \frac{1}{s(s^2+4s+4)} = \frac{1}{s}\,\frac{4}{(s+2)^2} \quad (1)$$

$$= \frac{A}{s} + \frac{B}{s+2} + \frac{C}{(s+2)^2} = \frac{4}{s(s+2)^2} \quad (1)$$

$$A(s+2)^2 + Bs(s+2) + Cs \equiv 4 \quad (1)$$

$$As^2 + 4As + 4A + Bs^2 + 2Bs + Cs \equiv 4$$

$$A + B = 0 \qquad B = -A = -1\cancel{\tfrac{1}{4}} \quad (1)$$

$$4A + 2B + C = 0 \qquad C = -\cancel{1 + \tfrac{1}{2} = -\tfrac{1}{2}} \; -2 \quad (1)$$

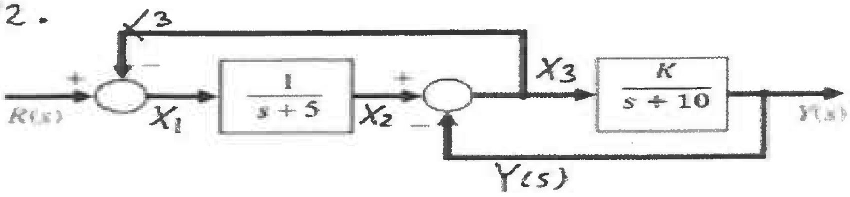
$$4A = 4 \qquad A = 1\cancel{\tfrac{1}{4}} \quad (1)$$

$$\frac{1}{4s} - \frac{1}{4(s+2)} - \frac{1}{2(s+2)^2} \Rightarrow \text{inverse Laplace}$$

$$x_2(t) = 1\cancel{\tfrac{1}{4}} - 1\cancel{\tfrac{1}{4}}\,e^{-2t} - 2\cancel{\tfrac{1}{2}}\,t\,e^{-2t} \quad (1)$$

[9]

Q2.



(a)

$$R(s) - X_3(s) = X_1(s) \quad ①$$

$$X_2(s) = \frac{1}{s+5} X_1(s) \quad ①$$

$$X_2(s) - Y(s) = X_3(s) \quad ①$$

$$Y(s) = \frac{K}{s+10} X_3(s) \quad ①$$

$$Y(s) = \frac{K}{s+10}(X_2 - Y) \Rightarrow$$

$$\left(1 + \frac{K}{s+10}\right) Y = \frac{K}{s+10} X_2$$

$$\frac{X_2}{Y} = \frac{1 + \frac{K}{s+10}}{\frac{K}{s+10}}$$

$$R + Y = (s+6)\left(\frac{1+\frac{K}{s+10}}{\frac{K}{s+10}}\right) Y$$

$$R = \left[(s+6)\frac{\left(1+\frac{K}{s+10}\right)}{\left(\frac{K}{s+10}\right)} - 1\right] Y$$

$$\frac{Y}{R} = \frac{1}{\frac{(s+6)\left(1+\frac{K}{s+10}\right)}{\left(\frac{K}{s+10}\right)} - 1} = \frac{\frac{K}{s+10}}{(s+6)\left(1+\frac{K}{s+10}\right) - \frac{K}{s+10}}$$

$$= \frac{\frac{K}{s+10}}{s+6+\frac{(s+6)K}{s+10} - \frac{K}{s+10}} = \frac{K}{(s+6)(s+10) + K(s+6) - K} \quad 1$$

$$= \frac{K}{s^2+16s+60+Ks+6K-K} = \frac{K}{s^2+(16+K)s+(60+5K)} \quad 1$$

[6]

(b)

(1d) $60+5K$ 1

$16+K$ $\quad 0$ 1

$b_1 = -\left(0 - \frac{(60+5K)(\cancel{16+K})}{\cancel{16+K}}\right)$

$60+5K$ $\quad 16+K>0 \quad K>-16$ 1

$0 \quad 60+5K>0 \quad K>-\frac{60}{5} = -12$

1 [5]

1 $\boxed{K>-12}$ for stability

(c.) $K=8 \quad \omega_n^2 = 100$ 1 $\omega_n = 10$ 1

$2\zeta\omega_n = 24$ 1 $\zeta = \frac{24}{20} = \frac{12}{10} = 1.2$ 1

$t_p = \frac{4}{12} = \frac{1}{3}\,S$ 1

[5]

$t_p = \frac{4}{\omega d}$

Root locus

$$\frac{K}{(S+5)(S+10)}$$

(1) No zeros (1)

(2) poles at $-5, -10$ (1)

(3) Two Asymptotes $np - nz = 2$ (1)

$$\sigma = \frac{\Sigma p - \Sigma z}{np - nz} = \frac{(-5-10)}{2} = -7.5$$

$$\theta = \pm \frac{(2k+1)\pi}{np - nz} = \pm \frac{(2k+1)\pi}{2} \to +\pi/2,\ \to -\pi/2$$ (1)

(4) Break away point

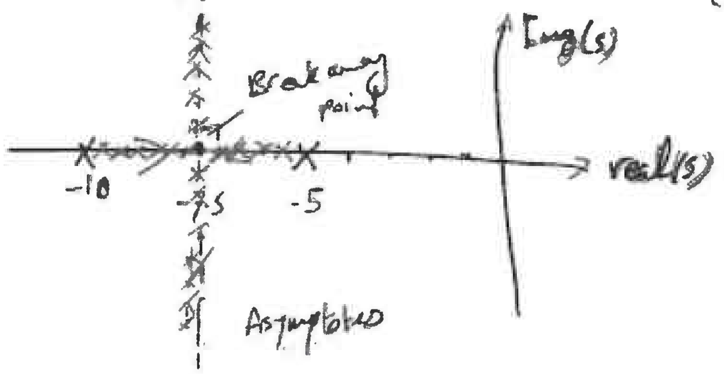
$N = 1 \quad N' = 0$

$d = S^2 + 15S + 50 \quad d' = 2S + 15$

$Nd - d'N = 0$ (1)

$(2S + 15) = 0$

$S = -7.5$



(3)

[9]

Q3)

(4) → Just drawing

(5) → For calculations and linking

Q3.

(a)

$$ess = lim_{s\to 0}\, s\frac{R(s)}{1+G(s)} = lim_{s\to 0}\, s\frac{1/s\hat{}2}{1+G(s)} = lim_{s\to 0}\frac{1/s}{1+\frac{K(s+2)}{s(s+1)(s+10)}} = \frac{10}{2K} = 0.01 \text{ find K}$$

① ① ① simplified

$2K = \frac{10}{0.01}$ ①

$K = \frac{1000}{2} = 500$

(b)

$$\frac{KG}{1+KG} = \frac{K(s+2)}{s(s+1)(s+10)+K(s+2)}$$

①

2 points for writing the characteristic polynomial

S=-5 subsititute in the denominator

K=100/3

(c) Bode plot magnitude and phase

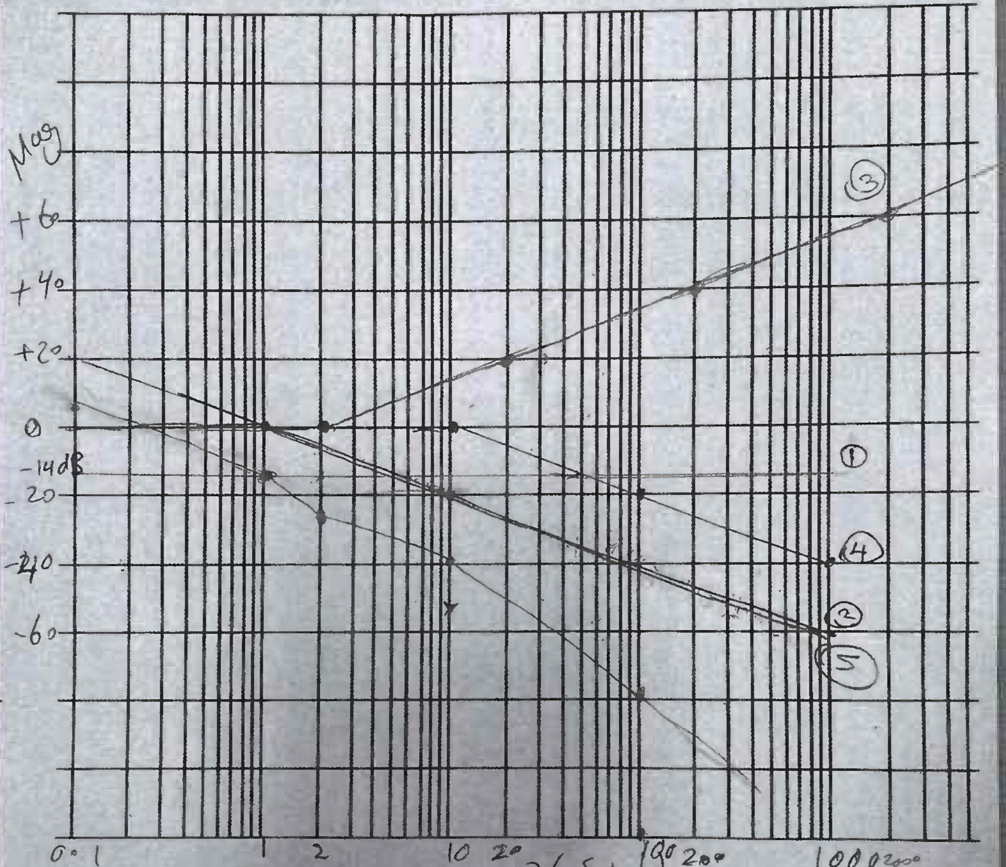
① point for replacing $s=-5$

① find value $K = \frac{100}{3}$

Mag plot

Mag $\frac{1}{s+1}$ $= 20\log_{10}\left(\frac{1}{s+1}\right) \rightarrow$ 5

WMBE024-05



$KG(s) = \frac{s+2}{s(s+1)(s+10)} \rightarrow \frac{2\left(\frac{s}{2}+1\right)}{10\,s(s+1)\left(\frac{s}{10}+1\right)}$ 5

Mag $20\log_{10}\left(\frac{2}{10}\right) = -13.9$ dB Constant line ----- 1 -13.9 dB 3

Mag $\frac{1}{s} \rightarrow 20\log_{10}(s)$ **END OF PAPER** 2 -20 dB

Mag $\left(\frac{s}{2}+1\right) + 20\log_{10}\left(\frac{s}{2}+1\right)$ 3

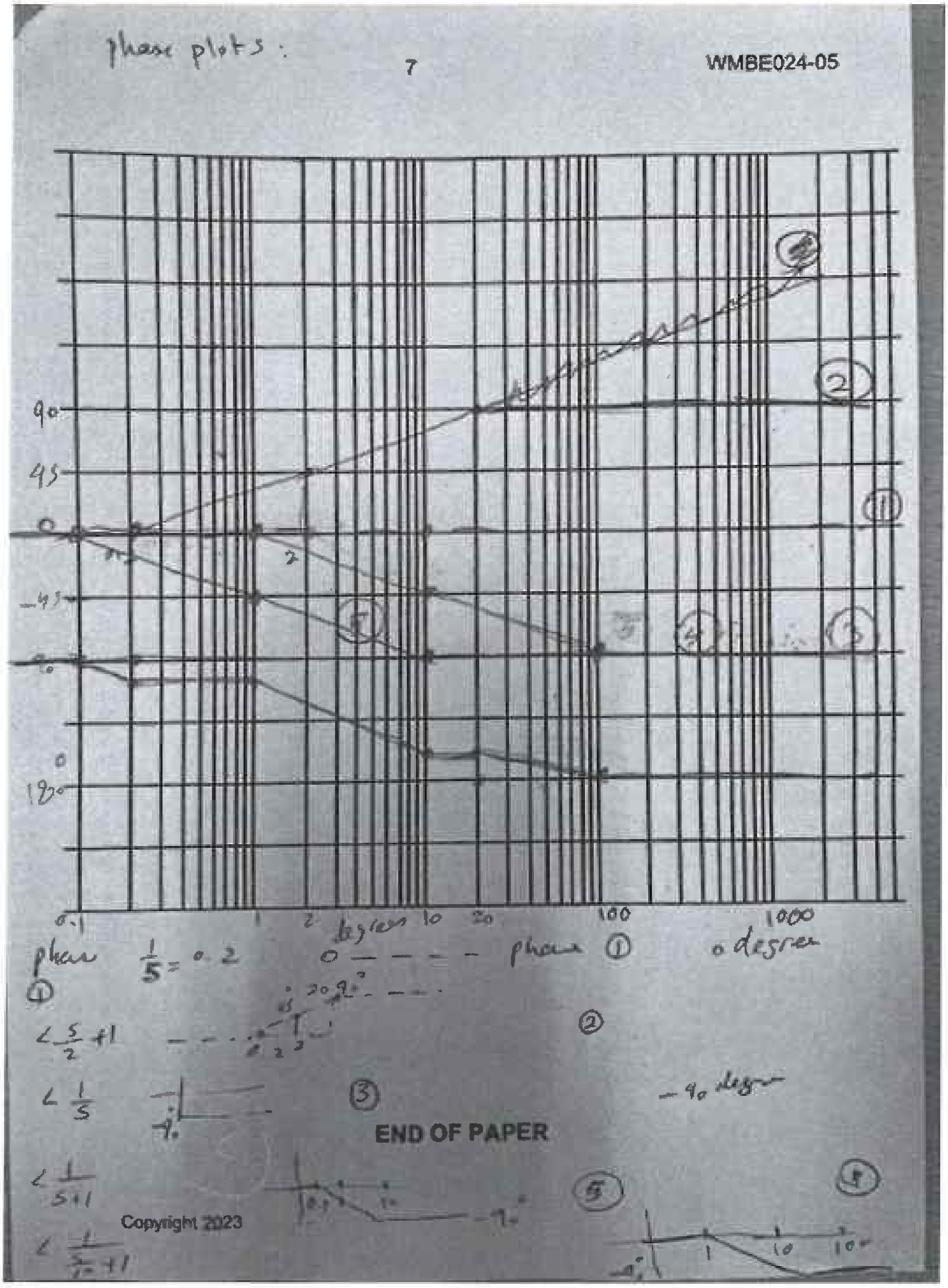
Mag $\left(\frac{s}{10}+1\right)$ 4

* 0.5 point per calculation per line 2.5

0.5 point per line that is drawn 2.5

for final line 3

Magnitude 8 marks



5

3

* 0.5 point per calculation per line. (2.5)
0.5 point per line that is drawn. (2.5)
3 marks for the final line. (3)

Phase (8) marks

Q4.

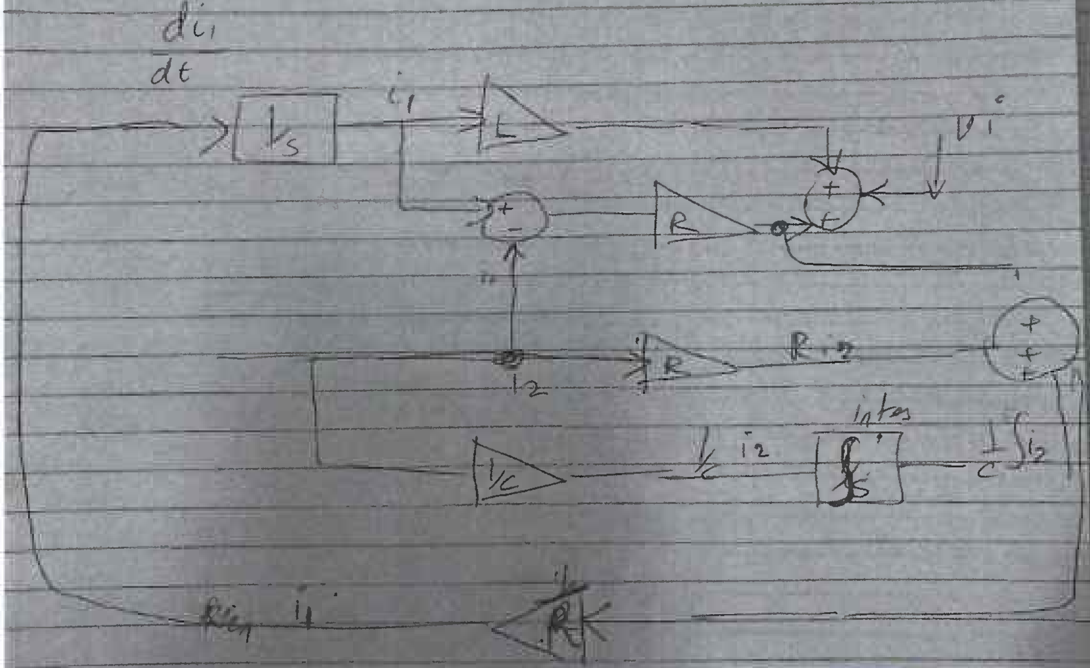
4) a

marks

$$L\frac{di_1}{dt} + R(i_1 - i_2) = v_i$$

$$Ri_2 + \frac{1}{c}\int i_2\,dt + R(i_2 - i_1) = 0$$

5



5

$\frac{10}{10}$

$$\dot{x} = Ax + Bu$$

4b)

$$\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \ddot{q}_1 \\ \ddot{q}_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -\frac{R_1}{L} & \frac{R_1}{L} \\ 0 & \frac{1}{CR_1} & \frac{R_1}{R_2} & -\frac{R_1}{R_2} \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \frac{v_i}{L} \\ 0 \end{bmatrix} v_i$$

$\frac{10}{10}$

4c) $y = Cx + Du$ 1

$V_o = R_2 i_2$ 1

$$V_o = R_2 \frac{dq_2}{dt}$$

$$V_o = \begin{bmatrix} 0 & 0 & 0 & R_2 \\ 1 & 1 & 1 & \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ \dot{q}_1 \\ \dot{q}_2 \end{bmatrix}$$ 3